2.1

Linear and Quadratic Functions and Modeling

Polynomial Function – Let *n* be a non-negative integer and let a_n, a_{n-1}, … a_1, a_0 be real numbers with $a_n \neq 0$ the function given by $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_0$ is a polynomial function of degree *n*.

Degree – the degree of a polynomial function is the degree of the polynomial in one variable that is the largest power of x that appears.

The **leading coefficient** is a_n.

Ex. $f(x) = 2x^5 - 4x^3 + x - 7$ has leading coefficient 2 and degree 5.

Polynomial Functions of No and Low Degree

Zero Function:	$f(x) = 0$	degree: undefined
Constant function:	$f(x) = a$, $a \neq 0$	degree: 0
Linear function:	$f(x) = ax + b$, $a \neq 0$	degree: 1
Quadratic function:	$f(x) = ax^2 + bx + c,\ a \neq 0$	degree: 2

Note: Vertical line is not a function. A line in the Cartesian plane is the graph of a linear function if and only if it is a slant line.

Another property that characterizes a linear function is its ***rate of change*.** The **average rate of change** of a function $y = f(x)$ between $x = a$ and $x = b$, $a \neq b$, is:

$$\frac{f(b) - f(a)}{b - a}$$ in a linear function also call **rate of change**.

Constant Rate of Change – a function defined on all numbers is a linear function if and only if it has a constant nonzero rate of change between any two points on its graph. (commonly called slope)

To find the equation of a linear function (y = mx +b).

1. Find slope.
2. Find the y-intercept

Ex. Write an equation for a linear function *f* such that *f*(-2) = 4 and *f*(3) = 1.

We must find the slope of the line through the points (-2, 4) and (3, 1).

$$m = \frac{1-4}{3-(-2)} = \frac{-3}{5}$$

Using the point slope formula $y - y_1 = m(x - x_1)$, we can find the equation of the line.

$$y - 4 = \frac{-3}{5}(x+2) \rightarrow y - 4 = \frac{-3}{5}x + \frac{-6}{5} \rightarrow y = \frac{-3}{5}x + \frac{14}{5}$$

Characterizing the Nature of a Linear Function

Point of View	**Characterization**
Verbal	polynomial of degree 1
Algebraic	$f(x) = mx + b$ ($m \neq 0$)
Graphical	slant line with slope *m* and *y*-intercept *b*
Analytical	function with constant nonzero rate of change *m*: *f* is increasing if *m* > 0, decreasing if *m* < 0; initial value of the function = *f*(0) = *b*

Linear Correlation and Modeling

When points of a scatter plot are clustered along a line, there is a

linear correlation between the quantities represented by the data.

Properties of the Correlation Coefficient, r

1. $-1 \leq r \leq 1$

2. When $r > 0$, there is a positive linear correlation

3. When $r < 0$, there is a negative linear correlation

4. When $|r| \approx 1$, there is a strong linear correlation.

5. When $r \approx 0$, there is weak or no linear correlation.

See graphs on pg. 175.

Regression Analysis

1. Enter and plot the data (scatter plot).

2. Find the regression model that fits the problem situation.

3. Superimpose the graph of the regression model on the scatter

plot, and observe the fit.

4. Use the regression model to make the predictions called for

in the problem.

Do example 8 on pg. 181 as a class. (Select Diagnostic On from Catalog)

Quadratic Function and their graphs

A quadratic function is a polynomial function of degree 2. The **standard quadratic form** for a parabola (polynomial function of degree 2) is: $f(x) = \boldsymbol{a}x^2 + \boldsymbol{b}x + \boldsymbol{c}$, $a \neq 0$

The **vertex form** of any quadratic function is: $\mathbf{f(x) = \boldsymbol{a}(x - \boldsymbol{h})^2 + \boldsymbol{k}}$

a determines the opening and the vertical shrink or stretch, $a > 0$ opens up and $a < 0$ opens down.

x = *h* is the axis of symmetry, (*h, k*) is the vertex

$$h = \frac{-b}{2a} \qquad\qquad k = c - ah^2$$

Also k = f(h).

To find zeros (x-intercepts or roots) of a quadratic function you may factor the expression and solve, complete the square then solve or use the quadratic formula.

To describe algebraically means tell the **vertex**, the **axis of symmetry**, the **x-int**. and how it **opens**.

Do examples 5 and 6 in class. Pg. 178-179.

Example from HW. #40,